

## The $r:r^2:r^3$ Theorem

Making ratio madness easy one ratio at a time...

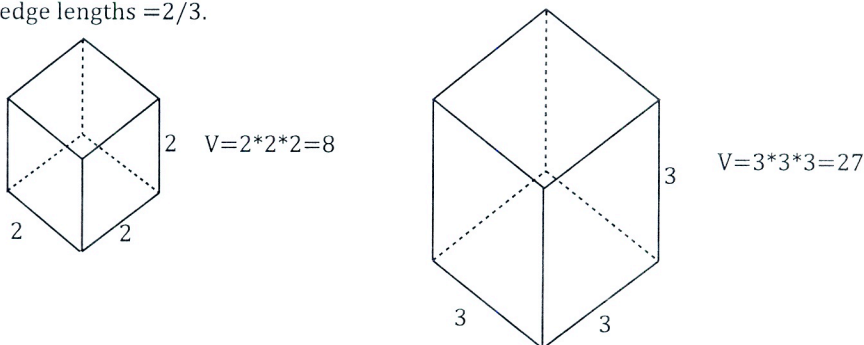
To show you what this is, let's start with a typical question... If two squares have a ratio of similarity of  $2/3$ , what is the ratio of their areas? What do you think? Two thirds? That's what most people say... but you're smarter than that, now aren't you? Let's take a look...

Ratio of the sides  $= 2/3$ .



That means the area ratio is actually  $4/9$  or  $(2/3)^2$ . That means that ratio of the areas of the two figures is the square of the ratio of their side lengths! But what about volume? Can you guess? Let's take a look anyway...

Ratio of the edge lengths  $= 2/3$ .



That means the volume ratio is actually  $8/27$ , or  $(2/3)^3$ . That means the ratio of the volumes of the two figures is the cube of the ratio of their edge (same as side) lengths! This means that the length ratios (side length, edge length, perimeter etc.) are to the first power, namely "r." Area, is squared, " $r^2$ ," and volume is cubed, " $r^3$ ." This should come as no surprise because 2in means two inches in length,  $2\text{in}^2$  is an area of two square inches, and  $2\text{in}^3$  is 2 cubic inches of volume!  $r:r^2:r^3$ ! Let's look at some examples of how to apply this...

Ex. 1. The ratio of the perimeters of two triangles is  $3/4$ . Find the ratio of their areas.

$$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

Ex. 2. The ratio of the areas of the faces of two prisms is  $16/25$ . Find the ratio of their volumes.

$$\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$$

For this kind, it's easier to find the length ratio "r" first, then find what you need from there.

Ex. 3. The ratio of the volumes of two prisms is  $8/27$ . Find the ratio of their edge lengths.

$$\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

$$\left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3} = \frac{64}{125}$$

Find each ratio...

1. The ratio of the sides of two triangles is  $\frac{4}{5}$ . Find the ratio of their areas.

3. The ratio of the perimeters of two parallelograms is  $\frac{3}{8}$ . Find the ratio of their areas.

5. The ratio of the volumes of two prisms is  $\frac{27}{64}$ . Find the ratio of their edge lengths.

7. The ratio of the perimeters of two squares is  $\frac{2}{7}$ . Find the ratio of their areas.

2. The ratio of the areas of two polygons is  $\frac{9}{49}$ . Find the ratio of their edge lengths.

4. The ratio of the edge lengths of two prisms is  $\frac{1}{3}$ . Find the ratio of their volumes.

6. The ratio of the edge lengths of two pyramids is  $\frac{7}{8}$ . Find the ratio of their volumes.

8. The ratio of the areas of the faces of two prisms is  $\frac{25}{64}$ . Find the ratio of their volumes.

Bubble all the correct answers from above. Don't bubble incorrect answers.

☐  $\frac{343}{512}$  ☐  $\frac{4}{49}$  ☐  $\frac{3}{4}$  ☐  $\frac{9}{121}$  ☐  $\frac{16}{49}$  ☐  $\frac{125}{144}$  ☐  $\frac{125}{512}$  ☐  $\frac{1}{27}$  ☐  $\frac{1}{36}$  ☐  $\frac{3}{9}$  ☐  $\frac{9}{64}$  ☐  $\frac{16}{25}$  ☐  $\frac{3}{7}$  ☐  $\frac{1}{8}$

9. The ratio of the perimeters of two triangles is  $5/9$ . Find the ratio of their areas.
10. The ratio of the volumes of two prisms is  $1/343$ . Find the ratio of their edge length.
11. The ratio of the areas of the faces of two prisms is  $25/36$ . Find the ratio of their volumes.
12. The ratio of the volumes of two prisms is  $64/729$ . Find the ratio of the areas of their faces.
13. The ratio of the volumes of two prisms is  $27/125$ . Find the ratio of the area of their faces.
14. The ratio of the perimeters of two trapezoids is  $5/11$ . Find the ratio of their areas.
15. The ratio of the areas of the faces of two prisms is  $27/512$ . Find the ratio of their volumes.
16. The ratio of the areas volumes two pyramids is  $125/1728$ . Find the ratio of the areas of their bases.

Bubble all the correct answers from above. Don't bubble incorrect answers.

☐  $16/25$   
 ☐  $25/144$   
 ☐  $25/49$   
 ☐  $25/121$   
 ☐  $125/512$   
 ☐  $16/81$   
 ☐  $9/64$   
 ☐  $25/81$   
 ☐  $16/25$   
 ☐  $16/12$   
 ☐  $1/7$   
 ☐  $216/512$   
 ☐  $125/216$   
 ☐  $1/8$